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Research Article

On Fuzzy *Sp*-Open Sets

Hakeem A. Othman[1, 2]

[1] *Department of Mathematics, Rada'a College of Education and Science, Albida, Yemen*
[2] *Department of Mathematics, Faculty of Science, King Khalid University, P.O. Box 9004, Abha, Saudi Arabia*

Correspondence should be addressed to Hakeem A. Othman, hakim_albdoie@yahoo.com

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A new class of generalized fuzzy open sets in fuzzy topological space, called fuzzy *sp*-open sets, are introduced, and their properties are studied and the relationship between this new concept and other weaker forms of fuzzy open sets we discussed. Moreover, we introduce the fuzzy *sp*-continuous (resp., fuzzy *sp*-open) mapping and other stronger forms of *sp*-continuous (resp., fuzzy *sp*-open) mapping and establish their various characteristic properties. Finally, we study the relationships between all these mappings and other weaker forms of fuzzy continuous mapping and introduce fuzzy *sp*-connected. Counter examples are given to show the noncoincidence of these sets and mappings.

1. Introduction

In 1996, Dontchev and Przemski, [1] have introduced the concept of *sp*-open sets in general topology. In this paper, we extend the notion of *sp*-open sets to fuzzy topology space and study some notions based on this new concept. We further study the relation between fuzzy *sp*-open sets and other types of fuzzy open sets. We also introduce the concepts of fuzzy *sp*-continuous (resp., fuzzy *sp*-open) mapping, other stronger forms of fuzzy *sp*-continuous (resp., fuzzy *sp*-open) mapping, and discuss their relation with other weaker forms of fuzzy continuous mapping.

2. Preliminaries

Throughout this paper, by (X, τ) or simply by X we mean a fuzzy topological space (fts, shorty) and $f : X \to Y$ means a mapping f from a fuzzy topological space X to a fuzzy topological space Y. If u is a fuzzy set and p is a fuzzy singleton in X, then $N(p)$, $\mathrm{Int}\,\lambda$, $\mathrm{cl}\,u$, u^c denote, respectively, the neighborhood system of p, the interior of u, the closure of u, and complement of u.

Now, we recall some of the basic definitions and results in fuzzy topology.

Definition 2.1 (see [2]). A fuzzy singleton p in X is a fuzzy set defined by: $p(x) = t$, for $x = x_0$ and $p(x) = 0$ otherwise, where $0 < t \le 1$. The point p is said to have support x_0 and value t.

Definition 2.2. A fuzzy set u in a fts X is called fuzzy α-open [3] (resp., Fuzzy preopen [4], Fuzzy β-open [5]) set if $u \le \mathrm{Int}\,\mathrm{cl}\,\mathrm{Int}\,u$ (resp., $u \le \mathrm{Int}\,\mathrm{cl}\,u$, $u \le \mathrm{cl}\,\mathrm{Int}\,\mathrm{cl}\,u$, $u \le \mathrm{cl}\,\mathrm{Int}\,u$). The family of all fuzzy α-open (resp., fuzzy preopen, fuzzy β-open, fuzzy semiopen) sets of X is denoted by $\mathrm{F\alpha O}(X)$ (resp., $\mathrm{FPO}(X)$, $\mathrm{F\beta O}(X)$, $\mathrm{FSO}(X)$).

Definition 2.3 (see [4]). Let u be any fuzzy set. Then,

(i) $\mathrm{pcl}\,u = \wedge\{v : v \ge u, v$ is a fuzzy preclosed set of $X\}$ is called preclosure,

(ii) $\mathrm{pInt}\,v = \vee\{v : v \le u, v$ is a fuzzy preopen set of $X\}$ is called pre-Interior.

The definitions of scl, sInt, α-cl, and α-Int are similar.

Theorem 2.4. *For any fuzzy set u in a* fts X*, the following statements are true:*

(i) $\mathrm{scl}\,u = u \vee \mathrm{Int}\,\mathrm{cl}\,u$ *and* $\mathrm{sInt}\,u = u \wedge \mathrm{cl}\,\mathrm{Int}\,u$ *[6],*

(ii) $\mathrm{pcl}\,u \ge u \vee \mathrm{cl}\,\mathrm{Int}\,u$ *and* $\mathrm{pInt}\,u \le u \wedge \mathrm{Int}\,\mathrm{cl}\,u$ *[7].*

Theorem 2.5 (see [3, 4]). *The arbitrary union of fuzzy preopen (resp., fuzzy semiopen) sets is a fuzzy preopen (resp., fuzzy semiopen) set.*

Definition 2.6. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be

(i) fuzzy α-continuous [3] if $f^{-1}(v)$ is fuzzy α-open set in X for each fuzzy open set v in Y,

(ii) fuzzy semi continuous [3] if $f^{-1}(v)$ is fuzzy semiopen set in X for each fuzzy open set v in Y,

(iii) fuzzy β-continuous [5] if $f^{-1}(v)$ is fuzzy β-open set in X for each fuzzy open set v in Y.

3. Fuzzy *SP*-Open Set

Definition 3.1. A fuzzy subset u of fuzzy space X is called fuzzy *sp*-open set if $u \le \operatorname{Int}\operatorname{cl} u \vee \operatorname{cl}\operatorname{Int} u$. The class of all fuzzy *sp*-open sets in X will be denoted be $\mathrm{FSP} - O(X)$.

It is obvious that $\mathrm{FPO}(X) \vee \mathrm{FSO}(X) \le \mathrm{FSP} - O(X) \le \mathrm{F}\beta\mathrm{O}(X)$.

Proposition 3.2. *Let u be fuzzy sp -open set such that* $\operatorname{Int} u = 0$. *Then, u is fuzzy preopen.*

Using Theorem 2.5, we can easily prove the next corollary.

Corollary 3.3. *Any union of fuzzy sp -open sets is a fuzzy sp-open set.*

Remark 3.4. The intersection of fuzzy *sp*-open sets need not be fuzzy *sp*-open set. This is illustrated by the following example.

Example 3.5. Let $X = \{a, b, c\}$ and v_1, v_2, v_3, and v_4 be fuzzy sets of X defined as

$$\begin{aligned} &v_1(a) = 0.0, \quad &v_1(b) = 0.0, \quad &v_1(c) = 0.4,\\ &v_2(a) = 0.9, \quad &v_2(b) = 0.6, \quad &v_2(c) = 0.0,\\ &v_3(a) = 0.0, \quad &v_3(b) = 0.3, \quad &v_3(c) = 0.4,\\ &v_4(a) = 0.9, \quad &v_4(b) = 0.7, \quad &v_4(c) = 0.2. \end{aligned} \tag{1}$$

Let $\tau = \{o_x, v_1, v_2, v_1 \vee v_2, 1_x\}$. Clearly, τ is a fuzzy topology on X, and by easy computation, it follows that v_3 and v_4 are fuzzy *sp*-open sets. But $(v_3 \wedge v_4)$ is not a fuzzy fuzzy *sp*-open set.

Theorem 3.6. *For any fuzzy subset u of a fuzzy space X, the following properties are equivalent:*

(i) *u is fuzzy sp-open,*

(ii) $u \ge \operatorname{pInt} u \vee \operatorname{sInt} u$.

Proof. $(i) \Rightarrow (ii)$. Let u be a fuzzy *sp*-open, that is, $u \le \operatorname{Int}\operatorname{cl} u \vee \operatorname{cl}\operatorname{Int} u$. Then, we have

$$\begin{aligned} \operatorname{pInt} u \vee \operatorname{sInt} u &\le (u \wedge \operatorname{Int}\operatorname{cl} u) \vee (u \wedge \operatorname{cl}\operatorname{Int} u)\\ &\le u \wedge (\operatorname{Int}\operatorname{cl} u \vee \operatorname{cl}\operatorname{Int} u)\\ &\le u \wedge u = u. \end{aligned} \tag{2}$$

□

Definition 3.7. A fuzzy subset u of fuzzy space X is called fuzzy *sp*-closed set if $\operatorname{Int}\operatorname{cl} u \wedge \operatorname{cl}\operatorname{Int} u \le u$. The class of all fuzzy *sp*-closed sets in X will be denoted be FSP-$C(X)$.

Definition 3.8. Let u any fuzzy set. Then,

(i) $sp\text{-}\operatorname{cl} u = \wedge\{v : v \ge u, v \text{ is a fuzzy } sp\text{-closed set of } X\}$ is called fuzzy *sp*-closure,

(ii) $sp\text{-}\operatorname{Int} v = \vee\{v : v \le u, v \text{ is a fuzzy } sp\text{-open set of } X\}$ is called fuzzy *sp*-Interior.

By using Definitions 3.1, 3.7, and 3.8, we can prove the following theorems.

Theorem 3.9. *Let u and v be the fuzzy sets in* fts X. *Then, the following statements hold*

(i) sp-$\operatorname{cl}(u)$ *is fuzzy sp-closed,*

(ii) $u \subseteq SP\text{-}C(X) \Leftrightarrow u = sp\text{-}\operatorname{cl}(u)$,

(iii) $u \le v \Rightarrow sp\text{-}\operatorname{cl}(u) \le sp\text{-}\operatorname{cl}(v)$,

(iv) sp-$\operatorname{Int} u$ *is fuzzy sp-open,*

(v) $u \subseteq SP\text{-}O(X) \Leftrightarrow u = sp\text{-}\operatorname{Int}(u)$,

(vi) $u \le v \Rightarrow sp\text{-}\operatorname{Int}(u) \le sp\text{-}\operatorname{Int}(v)$,

(vii) $\operatorname{Int} u \le sp\text{-}\operatorname{Int} u \le u \le sp\text{-}\operatorname{cl}(u) \le \operatorname{cl} u$.

Theorem 3.10. *For a fuzzy subset λ of a fuzzy space X, the following statements are holding:*

(i) $sp\text{-}\operatorname{cl} u \ge u \vee (\operatorname{Int}\operatorname{cl} u \wedge \operatorname{cl}\operatorname{Int} u)$,

(ii) $sp\text{-}\operatorname{Int} u \le u \wedge (\operatorname{Int}\operatorname{cl} u \vee \operatorname{cl}\operatorname{Int} u)$,

(iii) $sp\text{-}\operatorname{cl} u \ge \operatorname{scl} u \wedge \operatorname{pcl} u$,

(iv) $sp\text{-}\operatorname{Int} u \le \operatorname{sInt} u \vee \operatorname{pInt} u$.

Theorem 3.11. *For any fuzzy subset u of a fuzzy space X, the following statements are equivalent:*

(i) *u is fuzzy sp-closed,*

(ii) *u^c is fuzzy sp-open,*

(iii) $u \ge \operatorname{Int}\operatorname{cl} u \wedge \operatorname{cl}\operatorname{Int} u$, *and*

(iv) $u^c \le \operatorname{Int}\operatorname{cl} u \vee \operatorname{cl}\operatorname{Int} u$.

Theorem 3.12. *A fuzzy set u in a fuzzy topology space X is fuzzy sp -open if and only if for every fuzzy point $p \in u$, there exists a fuzzy sp -open set $v_p \le u$ such that $p \in v_p$.*

$$\begin{tikzcd}
 & \text{Fuzzy } \alpha\text{-open} \arrow[r] \arrow[d] & \text{Fuzzy preopen} \arrow[dr] \arrow[d] & \\
\text{Fuzzy open} \arrow[ur] \arrow[dr] & & & \text{Fuzzy } \beta\text{-open} \\
 & \text{Fuzzy semiopen} \arrow[r] & \text{Fuzzy } sp\text{-open} \arrow[ur] &
\end{tikzcd}$$

FIGURE 1

Proof. If u is a fuzzy *sp*-open set, then we may take $v_p = u$ for every $p \in u$.

Conversely, we have $u = \cup_{p\in u}\{p\} \leq \cup_{p\in u} v_p \leq u$ and, hence, $u = \cup_{p\in u} v_p$. This shows that u is a fuzzy *sp*-open set. □

From Definitions 2.2 and 3.1, the above "Implication Figure 1" illustrates the relation of different classes of fuzzy open sets.

Remark 3.13. The converse of these relations need not be true, in general as shown by the following examples.

Example 3.14. Let $X = \{a, b, c\}$ and v_1, v_2, v_3, and v_4 be fuzzy sets of X defined as

$$\begin{aligned} v_1(a) &= 0.5, & v_1(b) &= 0.3, \\ v_2(a) &= 0.5, & v_2(b) &= 0.6, \\ v_3(a) &= 0.6, & v_3(b) &= 0.3, \\ v_4(a) &= 0.6, & v_4(b) &= 0.7. \end{aligned} \tag{3}$$

Let $\tau = \{o_x, v_1, 1_x\}$. Clearly, τ is a fuzzy topological space on X, and by easy computation, we can see:

(i) v_2 is fuzzy *sp*-open set which is neither fuzzy α-open set nor fuzzy preopen,

(ii) v_3 is fuzzy *sp*-open which is not semiopen,

(iii) v_4 is fuzzy *sp*-open set which is not fuzzy open.

Example 3.15. Let $X = \{a, b, c\}$ and $v_1, v_2,$ *and* v_3 fuzzy sets of X defined as

$$\begin{aligned} v_1(a) &= 0.0, & v_1(b) &= 0.0, & v_1(c) &= 0.4, \\ v_2(a) &= 0.9, & v_2(b) &= 0.6, & v_2(c) &= 0.0, \\ v_3(a) &= 0.1, & v_3(b) &= 0.0, & v_3(c) &= 0.3. \end{aligned} \tag{4}$$

Let $\tau = \{o_x, v_1, 1_x\}$. Clearly, τ is a fuzzy topological space on X, and by easy computation, it follows that v_3 is fuzzy β-open set which is not fuzzy *sp*-open.

4. Fuzzy *SP*-Continuous Mapping

Definition 4.1. A mapping $f : X \to Y$ is said to be

(i) fuzzy *sp*-continuous if $f^{-1}(v)$ is fuzzy *sp*-open set in X for each fuzzy open set v in Y,

(ii) fuzzy $sp^{\star}$-continuous if $f^{-1}(v)$ is fuzzy *sp*-open set in X for each fuzzy *sp*-open set v in Y,

(iii) fuzzy $sp^{\star\star}$-continuous if $f^{-1}(v)$ is fuzzy open set in X for each fuzzy *sp*-open set v in Y.

Theorem 4.2. *For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(i) *f is fuzzy sp-continuous;*

(ii) *for every fuzzy singleton p in X and every open set v in Y such that $f(p) \subseteq v$, there exists a fuzzy sp -open set $u \subseteq X$ such that $p \subseteq u$ and $u \leq f^{-1}(v)$;*

(iii) *for every fuzzy singleton p in X and every open set v in Y such that $f(p) \subseteq v$, there exists a fuzzy sp -open set $u \subseteq X$ such that $p \subseteq u$ and $f(u) \leq v$;*

(iv) *the inverse image of each fuzzy closed set in Y is fuzzy sp-closed;*

(v) $\operatorname{Int}\operatorname{cl}\ (f^{-1}(v)) \wedge \operatorname{cl}\operatorname{Int}\ (f^{-1}(v)) \leq (f^{-1}(\operatorname{cl} v))$ *for each $v \subseteq Y$;*

(vi) $f[\operatorname{Int}\operatorname{cl}(u) \wedge \operatorname{cl}\operatorname{Int}(u)] \leq \operatorname{cl} f(u)$ *for each $u \subseteq X$.*

Proof. (i)⇒(ii) Let fuzzy singleton p be in X and every open set v in Y such that $f(p) \subseteq v$, there exists a fuzzy open set m be in Y such that $f(p) \leq m \leq v$. Since f is *sp*-continuous, $u = f^{-1}(m)$ is fuzzy *sp*-open and we have $p \leq f^{-1}(f(p)) \leq f^{-1}(m) \leq f^{-1}(v)$ or $p \leq u = f^{-1}(m) \leq f^{-1}(v)$.

(ii)⇒(iii) Let fuzzy singleton p be in X and every fuzzy open set v be in Y such that $f(p) \subseteq v$, there exists a fuzzy *sp*-open u such that $p \leq u$ and $u \leq f^{-1}(v)$. So, we have $p \leq u$ and $f(u) \leq f^{-1}((f(v)) \leq v$.

(iii)⇒(i) Let v be a fuzzy open set in Y and let us take $p \leq f^{-1}(v)$. This shows that $f(p) \leq f(f^{-1}(v)) \leq v$. Since v is a fuzzy open set, then there exists a fuzzy *sp*-open set u such that $p \leq u$ and $f(u) \leq v$. This shows that $p \leq u \leq f^{-1}(f(u)) \leq f^{-1}(v)$. By Theorem 3.12, it follows that $f^{-1}(v)$ is fuzzy *sp*-open set in X and hence f is fuzzy *sp*-continuous.

(i)⇒(iv) Let v be a fuzzy closed in Y. This implies that $I_Y - v$ is fuzzy open set. Hence, $f^{-1}(I_Y - v)$ is fuzzy *sp*-open set in X, that is, $(I_X - f^{-1}(v))$ is fuzzy *sp*-open set in X. Thus, $f^{-1}(v)$ is a fuzzy *sp*-closed set in X.

(iv)⇒(v) Let $v \subseteq Y$, then $f^{-1}(\operatorname{cl}\ v)$ is *sp*-closed in X, that is, $\operatorname{Int}\operatorname{cl}(f^{-1}(v)) \wedge \operatorname{cl}\operatorname{Int}(f^{-1}(v)) \leq \operatorname{Int}\operatorname{cl}(f^{-1}(\operatorname{cl} v)) \wedge \operatorname{cl}\operatorname{Int}(f^{-1}(\operatorname{cl} v)) \leq f^{-1}(\operatorname{cl}\ v)$.

(v)⇒(vi) Let $u \subseteq X$, put $v = f(u)$ in (v), then $\operatorname{Int}\operatorname{cl}(f^{-1}(f(u))) \wedge \operatorname{cl}\operatorname{Int}(f^{-1}(f(u))) \leq f^{-1}(\operatorname{cl}(f(u)))$ so that $\operatorname{Int}\operatorname{cl}\ (u)) \wedge \operatorname{cl}\operatorname{Int}\ (u) \leq f^{-1}(\operatorname{cl}(f(u)))$. This gives $f[\operatorname{Int}\operatorname{cl}\ (u) \wedge \operatorname{cl}\operatorname{Int}\ (u)] \leq \operatorname{cl} f(u)$.

(vi)⇒(i) Let $v \subseteq Y$, be fuzzy open set. put $u = I_Y - v$ and $u = f^{-1}(v)$, then $f[\operatorname{Int}\operatorname{cl}(f^{-1}(v)) \wedge \operatorname{cl}\operatorname{Int}(f^{-1}(v))] \leq \operatorname{cl}\ f(f^{-1}(v)) \leq \operatorname{cl}(v) = v$, that is, $f^{-1}(v)$*sp*-closed set in X, so f is *sp*-continuous mapping. □

Fuzzy sp^{**}-continuous $\longrightarrow$ Fuzzy sp^{*}-continuous $\longrightarrow$ Fuzzy sp-continuous

Fuzzy sp^{**}-openness $\longrightarrow$ Fuzzy sp^{*}-openness $\longrightarrow$ Fuzzy sp-openness

Figure 2

$$\begin{tikzcd}
\text{Fuzzy } \alpha\text{-continuous} \arrow[dr] & & \\
& \text{Fuzzy } sp\text{-continuous} \arrow[r] & \text{Fuzzy } \beta\text{-continuous} \\
\text{Fuzzy semicontinuous} \arrow[ur] & &
\end{tikzcd}$$

Figure 3

Definition 4.3. A mapping $f : X \to Y$ is said to be

(i) fuzzy *sp*-open (Fuzzy *sp*-closed) if $f(u)$ is fuzzy *sp*-open (fuzzy *sp*-closed) set in Y for each fuzzy open (fuzzy closed) set u in X,

(ii) fuzzy $sp^{\star}$-open (Fuzzy $sp^{\star}$-closed) if $f(u)$ is fuzzy *sp*-open (fuzzy *sp*-closed) set in Y for each fuzzy sp-open (fuzzy *sp*-closed) set u in X,

(iii) fuzzy $sp^{\star\star}$-open (Fuzzy $sp^{\star\star}$-closed) if $f(u)$ is fuzzy open (fuzzy closed) set in Y for each fuzzy *sp*-open (fuzzy *sp*-closed) set u in X.

Remark 4.4. If $f : X \to Y$ is fuzzy *sp*-continuous mapping and $g : Y \to Z$ is fuzzy *sp*-continuous mapping, then $gof : X \to Z$ may not be a fuzzy *sp*-continuous mapping; this can be show by the following example.

Example 4.5. Let $X = \{a, b, c\}$ and v_1, v_2, v_3, v_4 and v_5 be fuzzy sets of X defined as,

$$\begin{aligned}
&v_1(a) = 0.4, && v_1(b) = 0.6, && v_1(c) = 0.5,\\
&v_2(a) = 0.6, && v_2(b) = 0.4, && v_2(c) = 0.4,\\
&v_3(a) = 0.4, && v_3(b) = 0.6, && v_3(c) = 0.6,\\
&v_4(a) = 0.7, && v_4(b) = 0.7, && v_4(c) = 0.8,\\
&v_5(a) = 0.6, && v_5(b) = 0.4, && v_5(c) = 0.6.
\end{aligned} \tag{5}$$

Consider, ftsτ_1, τ_2, and τ_3 where $\tau_1 = \{o_x, v_1, v_2, v_1 \cap v_2, v_1 \cup v_2, 1_x\}$, $\tau_2 = \{o_x, v_4, 1_x\}$, and $\tau_3 = \{o_x, v_3, 1_x\}$ and the mapping $f : (X, \tau_1) \to (Y, \tau_2)$ and $g : (X, \tau_2) \to (Y, \tau_3)$ defined as $f(a) = b, f(b) = a, f(c) = c$ and $g(a) = a, g(b) = b, g(c) = c$. It is clear that f and g are fuzzy *sp*-continuous mapping. But $gof : (X, \tau_1) \to (Y, \tau_3)$ is not a fuzzy *sp*-continuous mapping. This because $(gof)^{-1}(v_3) = v_5$ and v_5 is not fuzzy *sp*-open set, and hence gof is not fuzzy *sp*-continuous mapping.

Theorem 4.6. *If $f : X \to Y$ is fuzzy sp -continuous mapping and $g : Y \to Z$ is fuzzy continuous mapping, then $gof : X \to Z$ is fuzzy sp-continuous mapping.*

Proof. Let v be a fuzzy set of Z. Then, $(gof)^{-1}(v) = f^{-1}(g^{-1}(v))$. And because g is fuzzy continuous this implies that $g^{-1}(v)$ is a fuzzy open set of Y and hence $f^{-1}(g^{-1}(v))$ is a fuzzy *sp*-open set in X. Therefore, gof is a fuzzy *sp*-continuous mapping. □

From Definitions 4.1 and 4.3, we can have the above "Implication Figure 2" illustrates the relation between different classes of fuzzy *sp*-continuous (fuzzy semi *sp*-open) mappings.

The above "Implication Figure 3" illustrates the relation between fuzzy *sp*-continuous and different classes of fuzzy continuous mapping.

Remark 4.7. We can see the converse of these relations need not be true, in general as shown by the following examples.

Example 4.8. Let $X = \{a, b, c\}$ and v_1, v_2, v_3, and v_4 fuzzy sets of X defined as

$$\begin{aligned}
&v_1(a) = 0.4, && v_1(b) = 0.6, && v_1(c) = 0.5,\\
&v_2(a) = 0.4, && v_2(b) = 0.4, && v_2(c) = 0.4,\\
&v_3(a) = 0.6, && v_3(b) = 0.4, && v_3(c) = 0.6,\\
&v_4(a) = 0.7, && v_4(b) = 0.7, && v_4(c) = 0.8.
\end{aligned} \tag{6}$$

Consider fts τ_1, τ_2, and τ_3 where $\tau_1 = \{o_x, v_1, v_2, v_1 \cap v_2, v_1 \cup v_2, 1_x\}$, $\tau_2 = \{o_x, v_4, 1_x\}$, and $\tau_3 = \{o_x, v_3, 1_x\}$ and the mapping $f : (X, \tau_1) \to (Y, \tau_2)$ and $g : (X, \tau_1) \to (Y, \tau_3)$ defined as $f(a) = a$, $f(b) = b$, $f(c) = c$ and $g(a) = a$, $g(b) = b$, $g(c) = c$. It is clear that:

(i) f is a fuzzy *sp*-continuous mapping which is neither fuzzy α-continuous mapping nor fuzzy semi continuous mapping,

(ii) g is a fuzzy β-continuous mapping which is not fuzzy *sp*-continuous mapping,

(iii) f is a fuzzy *sp*-continuous mapping which is neither fuzzy $sp^{\star}$-continuous mapping nor fuzzy $sp^{\star\star}$-continuous mapping, and this is because v_3 is fuzzy *sp*-open in τ_2 which is neither fuzzy *sp*-open nor fuzzy open in τ_1,

(iv) if $h : (X, \tau_3) \to (Y, \tau_1)$ defined as: $h(a) = a$, $h(b) = b$, $h(c) = c$, it is clear that h is a fuzzy *sp*-open mapping which is neither fuzzy $sp^{\star}$-open mapping nor fuzzy $sp^{\star\star}$-open mapping.

Definition 4.9. A fuzzy set u in an fts X is said to be fuzzy connected if u cannot be expressed as the union of two fuzzy separated sets.

Now, we can generalize the definition of fuzzy connected to define fuzzy *sp*-connected as follows.

Definition 4.10. A fuzzy set v in a fts (X, τ) is said to be fuzzy *sp*-connected if and only if v cannot be expressed as the union of two fuzzy *sp*-separated sets.

Theorem 4.11. *Let $f : X \to Y$ be a fuzzy sp -continuous surjective mapping. If v is a fuzzy sp -connected subset in X then, $f(v)$ is fuzzy connected in Y.*

Proof. Suppose that $f(m)$ is not connected in Y. Then, there exist fuzzy separated subsets u and v in Y such that $f(m) = u \cup v$.

Since f is fuzzy *sp*-continuous surjective mapping, $f^{-1}(u)$ and $f^{-1}(v)$ are fuzzy *sp*-open set in X and $m = f^{-1}(f(m)) = f^{-1}(u \cup v) = f^{-1}(u) \cup f^{-1}(v)$.

It is clear that $f^{-1}(u)$ and $f^{-1}(v)$ are fuzzy *sp*-separated in X. Therefore, m is not fuzzy *sp*-connected in X, which is a contradiction!!

Hence, Y is fuzzy connected. □

References

[1] J. Dontchev and M. Przemski, "On the various decompositions of continuous and some weakly continuous functions," *Acta Mathematica Hungarica*, vol. 71, no. 1-2, pp. 109–120, 1996.

[2] M. H. Ghanim, E. E. Kerre, and A. S. Mashhour, "Separation axioms, subspaces and sums in fuzzy topology," *Journal of Mathematical Analysis and Applications*, vol. 102, no. 1, pp. 189–202, 1984.

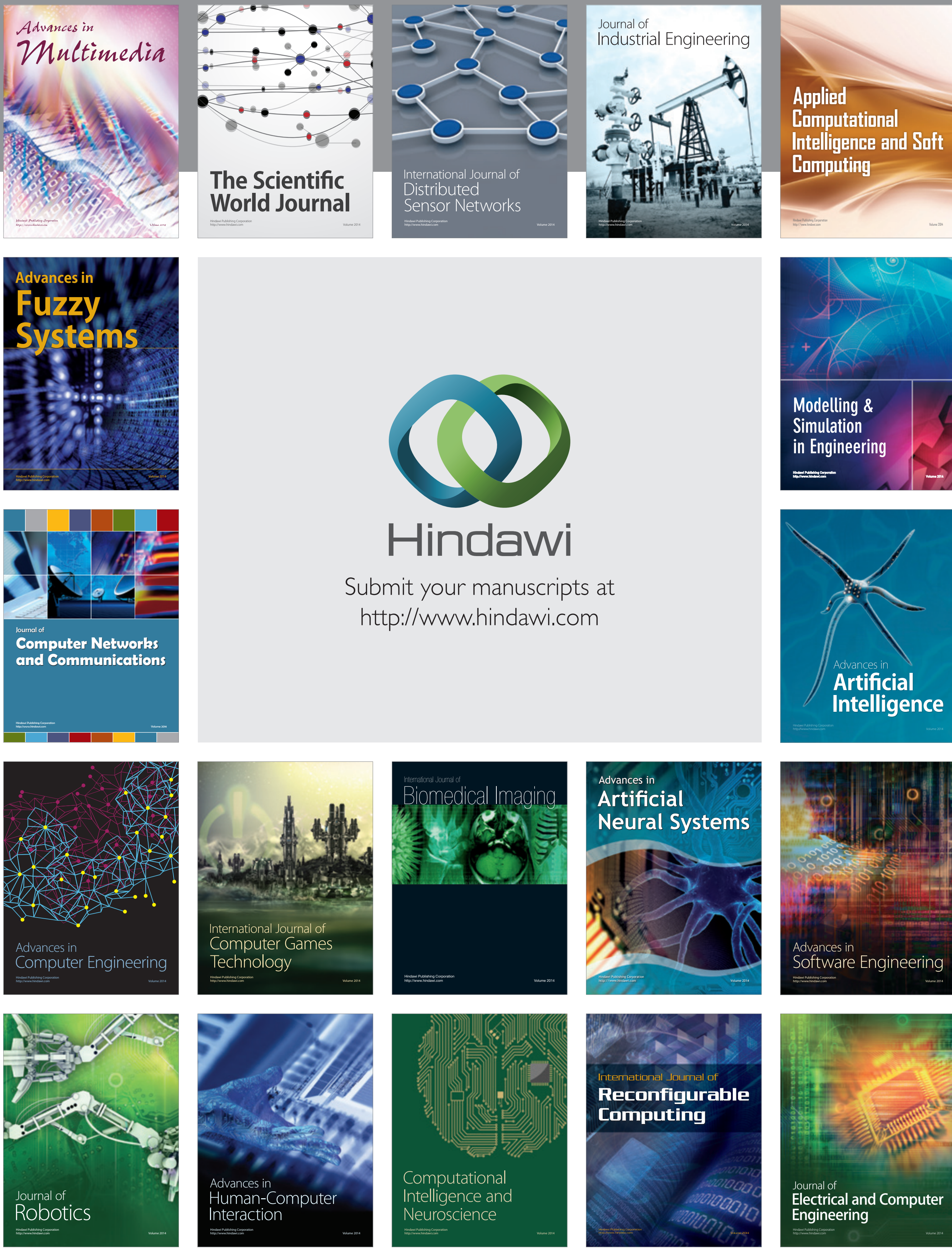
[3] A. S. Bin Shahna, "On fuzzy strong semicontinuity and fuzzy precontinuity," *Fuzzy Sets and Systems*, vol. 44, no. 2, pp. 303–308, 1991.

[4] M. K. Singal and N. Prakash, "Fuzzy preopen sets and fuzzy preseparation axioms," *Bulletin of Calcutta Mathematical Society*, vol. 78, pp. 57–69, 1986.

[5] A. S. Mashhour, M. H. Ghanim, and M. A. Fath Alla, "On fuzzy non continuous Mapping," *Bulletin of the Korean Mathematical Society*, vol. 78, pp. 57–69, 1986.

[6] B. Krsteska, "A note on the article "Fuzzy less strongly semiopen sets and fuzzy less strong semicontinuity"," *Fuzzy Sets and Systems*, vol. 107, no. 1, pp. 107–108, 1999.

[7] B. Kresteska, "Fuzzy strongly preopen sets and fuzzy strongly precontinuity," *Matematički Vesnik*, vol. 50, pp. 111–123, 1998.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